Local Two-Stage Myopic Dynamics for Network Formation Games

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Outline

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Objectives

Goal: Design *intuitive* dynamics that converge to good equilibria of Network Formation Games

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- Data and transportation networks

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- Pairwise Nash Stability
- Strong Stability

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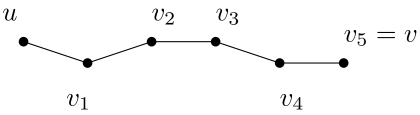
Examples:

- The Internet at the ISP level
- Mobile ad-hoc networks

Cost of a Network

Assume G connected

- $\gamma(u, v; G)$: cost of sending one packet from u to v



$$\gamma(u, v; G) = \sum_{i=1}^{5} c_{v_i}$$

- Maintenance cost per edge of $2\beta > 0$

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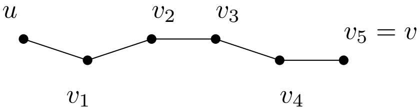

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Cost of network G:

$$\Gamma(G) = 2\beta|E| + \sum_{u,v} \gamma(u, v; G)$$

Who Should Pay?
Allocation Rules

- Cost of edge evenly split among endpoints.
- For routing cost, two allocation rules:
  1. Sender based allocation rule Y_d
  2. Flow based allocation rule Y_f

Cost to node u:

$$C_d(u;G) = \beta d_u(G) + Y_d(u;G) = \beta d_u(G) + \sum_{v \neq u} \gamma(u,v;G)$$

$$C_f(u;G) = \beta d_u(G) + Y_f(u;G) = \beta d_u(G) + c_u f(u;G)$$

$f(u;G)$ traffic forwarded or received by u.
Note: routing policy given.

Myerson Announcement Game

Nodes are selfish agents.

Edges result from bilateral agreements:

- node u announces desired neighborhood

$$S_u \subseteq V \setminus \{u\}$$

- $uv = e \in E$ if and only if $u \in S_v$ and $v \in S_u$.

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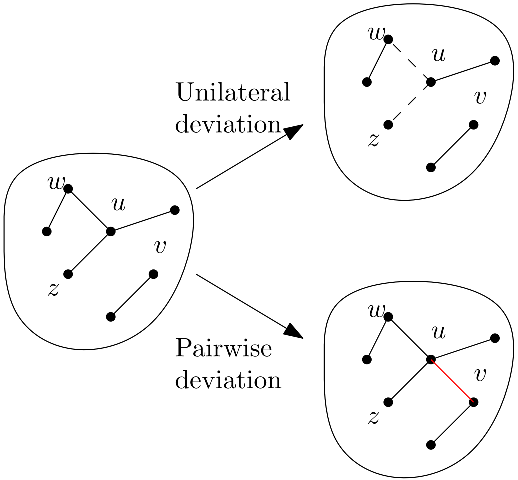
Network results from strategic interactions between selfish agents.

Deviations Considered



Nash network: no profitable unilateral deviation

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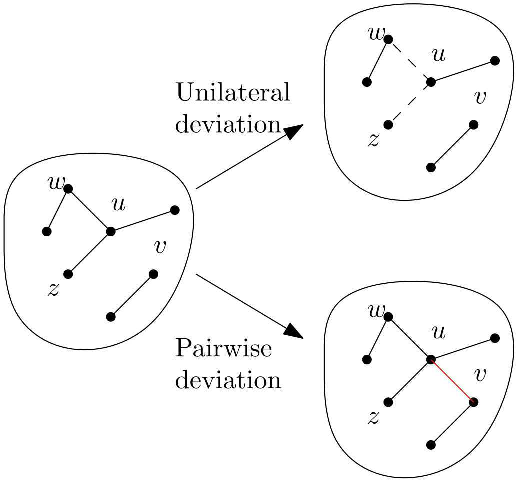

Nash network: no profitable unilateral deviation

Pairwise Nash stable network: both types of deviations

Why do we need dynamics?

Let $V_{\min} = \{u \in V | \forall v,\ c_u \leq c_v\}$.
Assume that, for all u, $c_u = \Theta(1)$. Then

Theorem (Efficient Networks)

For $\beta < c_{\min}$, the complete graph is the only efficient network.
For $\beta > c_{\min}$ only stars centered at a node $u \in V_{\min}$ are efficient.

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For all $\beta > 0$, all trees are PNS when using Y_f.
For all $\beta > n^2 c_{\max}$, all trees are PNS when using Y_d.

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Price of anarchy is linear in n.

Two-Stage Local Dynamics

Let $\ell > 1$ be given. Let G be the network topology. Select an active node u. Then

- u performs *two* consecutive deviations in a round (called *stages*) with nodes in his ℓ-neighborhood:
  - *First stage*: unilateral deviation
  - *Second stage*: *any* type of deviation

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  u minimizes its cost *at the end of the round*.
- All other nodes minimize their cost stage by stage.

Why These Two-Stage Dynamics?

- Simple generalization of best-response dynamics;
- only require *u* to know its ℓ-neighborhood; and
- Allows for links to be added and removed in one round.
- "one step" look-ahead type of dynamics

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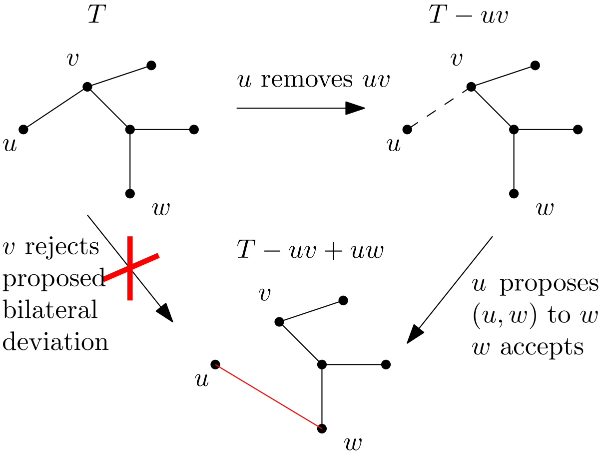
  $\hookrightarrow$ Allows node *u* to create a favorable intermediate state so that *w* accepts *u*'s offer *even* if *w*'s cost increases overall.

Why the "one-step look-ahead"? Intuition

Here $C(w;T) < C(w;T - uv + uw)$ and $C(u;T) > C(u;T - uv + uw)$.



Sender Allocation Rule Theorem

Assume that, for all u, $c_u = \Theta(1)$. Further, let $\beta > n^2 c_{\max}$ and $G^{(0)}$ be a connected network. Then

- the dynamics converge almost surely;
- all fixed points of the dynamics:
  1. have constant diameter; and
  2. are pairwise Nash stable.

Note: constant diameter implies constant efficiency ratio

our dynamics *select* good equilibria!

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But, can we do better?

Homogeneous Agents Setting
Preliminary Theorem

Assume that, for all u, $c_u = 1$. Further, let $\beta > n^2$ and $G^{(0)}$ be a connected network. Then

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But, what about that strong assumption on β?

Generalized Sender Based Setting Model

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Example (Connections Model)

Assume $g(x) = \alpha^x$, then we recover Jackson and Wolinsky's connections model.

Generalized Sender Based Setting
Strong Stability

A network G is *strongly stable* if no coalition of nodes can profitably deviate from G.
We still assume β sufficiently large for redundant links not to be valuable.
Then, for any strictly increasing function g,

- all line networks are strongly stable; and
- all star networks are strongly stable.

the price of anarchy and the price of stability under both strong stability and PNS are the same!

Generalized Sender Based Setting
Convergence Theorem

We still assume β sufficiently large for redundant links not to be valuable.

For two general classes of functions g,

- the dynamics converge almost surely;
- all fixed points of the dynamics are efficient; and
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Generalized Sender Based Setting Convergence Theorem

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Note that both $g(x) = \alpha^x$ (connections model with $\alpha > 1$) and $g(x) = x$ (Corbo and Parkes model) satisfy *both* conditions for the dynamics to converge.

Important Extension
“max” Function

Assume the cost to a node u in G is

$$C(u;G) = \beta d_u(G) + \max_{v \in V} \{d(u,v;G)\},$$

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Important Extension “max” Function

Assume the cost to a node u in G is

$$C(u;G) = \beta d_u(G) + \max_{v \in V} \{d(u,v;G)\},$$

and β is large enough for redundant links not to be valuable.

- The price of anarchy and the price of stability under both PNS and strong stability are identical;
- the dynamics converge almost surely;
- the limit networks are strongly stable and of diameter at most three.

The price of anarchy of the dynamics is $3/2$.

Flow Allocation Rule
Model and Results

Given $\beta > 0$, all trees are fixed points of our dynamics: we revise the utility model as in WINE'07 and WINE'08

$\hookrightarrow$ we allow utility transfers through contracts

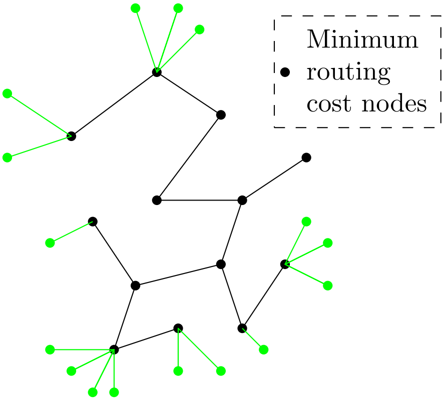
Under some reasonable assumptions about the utility transfers,

- the dynamics converge almost surely to;
  1. PNS networks (sometimes with "good" efficiency); and
  2. to the most efficient PNS network if unique.

our dynamics *sometimes* select good equilibria.

Selecting "Good" Networks

What happens if we have several nodes of minimum routing cost?



In the limiting state, all traffic is routed by minimum routing cost nodes.

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For flow based allocation rule Y_f,

- no restriction on $\beta > 0$;
- we *sometimes* select good equilibria;
- we select the most efficient PNS network if unique; but
- we require utility transfers between nodes.