Network Formation: Bilateral Contracting and Myopic Dynamics

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Motivation

Model
Utility
Contracting
Solution Concept
Dynamics

Results
Assumptions
Main Theorems
Efficiency
Interpretation

Conclusion
Future Work

Outline

Motivation

Model

Results

Conclusion

Objectives

Goal: Design *intuitive* dynamics that converge to "good" equilibria of Network Formation Games

Setting:

- Data networks
- Contracting
- Pairwise Stability

Examples:

- The Internet at the ISP level
- Mobile ad-hoc Networks

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Model
Utility
Contracting
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Conclusion
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Utility Model

For node $i \in V$, sum of three terms:

- Maintenance cost per edge of $\pi > 0$
- Routing cost of $c_i \geq 0$ per packet forwarded or received
- Disconnectivity cost of $\lambda > 0$ per unreachable node

Notation: cost to i in network topology G is $C_i(G)$

Motivation

Model
Utility
Contracting
Solution Concept
Dynamics

Results
Assumptions
Main Theorems
Efficiency
Interpretation

Conclusion
Future Work

Contracting

Edges in G result from *contracts* between nodes

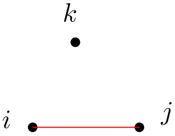
- common business tool
- captures current value of link

Contract (i, j): utility transfer of $Q(i, j; G)$ from i to j

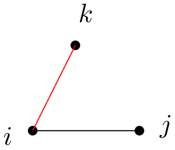
Example: Rubinstein Bargaining

Why Contracting?

Contracting induces payment *that remains fixed* until re-negotiation of contract.



i pays $Q(i, j; G + ij)$ to j



i pays $Q(i, j; G + ij)$ to j
and
$Q(i, k; G + ij + ik)$ to k

Payment Matrix, Contracting Graph and Total Utility

We keep track of

- payments in a *payment matrix* P;
- contracts in a *contracting graph* Γ

Thus the state of the network is given by the network topology G, the contracting graph Γ and the payment matrix P, and the total utility to node i is

$$U_i(G,P) = \sum_{j \neq i} (P_{ji} - P_{ij}) - C_i(G)$$

Motivation
Model
Utility
Contracting
Solution Concept
Dynamics
Results
Assumptions
Main Theorems
Efficiency
Interpretation
Conclusion
Future Work

Static Game
Pairwise Stability

One-shot static game.

- Each node selects nodes to propose contracts to; and
- selects nodes it accepts contracts from.
- Successful contract induces link.

Let G be the resulting topology.

- We set $P_{ij} = Q(i, j; G)$ if $(i, j) \in \Gamma$, and zero otherwise.

Definition (Pairwise Stability)

An outcome of the game is pairwise stable if it is a N.E. and no two players can benefit from a bilateral deviation.

Note: We only update the payments of the contracts involved in the deviation.

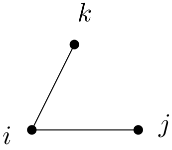
Motivation

Model
Utility
Contracting
Solution Concept
Dynamics

Results
Assumptions
Main Theorems
Efficiency
Interpretation

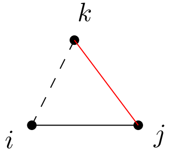
Conclusion
Future Work

Deviation Example



i pays $Q(i,j;G)$ to j

i pays $Q(i,k;G)$ to k



i still pays $Q(i,j;G)$ to j

k pays $Q(k,j;G-ik+kj)$ to j

Assume that *k* and *j* jointly deviate. *k* removes all contracts with *i*, and proposes (k,j) to *j*, and *j* accepts. Note that the payment from *i* to *j* did not change

Two-Stage Dynamics

A node *u* first unilaterally deviates with respect to some edge *uv* (stage 1), and then bilaterally deviates with some node *w* chosen by *u* (stage 2)

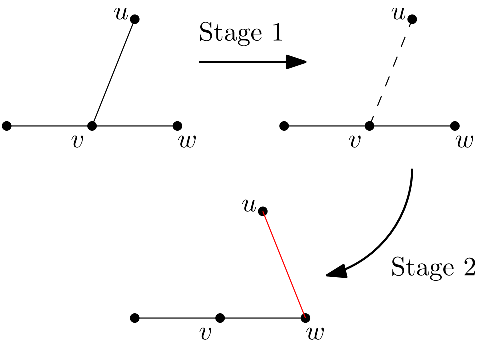
Why two stage dynamics?

"Unilateral deviation increases bargaining power".

$\hookrightarrow$ Allows node *u* to create a favorable intermediate state so that *w* accepts *u*'s offer *even* if *w*'s utility decreases.

Motivation
Model
Utility
Contracting
Solution Concept
Dynamics
Results
Assumptions
Main Theorems
Efficiency
Interpretation
Conclusion
Future Work

Example



Here one can see that, in all likelihood, *w*'s utility at the end of the round is lower than at the beginning.

Assumptions and Convergence

- Disconnectivity cost large enough to ensure connectivity
- Contracting function is
  - monotone and
  - anti-symmetric.

Definition (Convergence)

Given any initial outcome of the static game, we say the dynamics *converge* if, almost surely, there exists K such that, for $k > K$

$$\left(G^{(k+1)}, \Gamma^{(k+1)}, P^{(k+1)}\right) = \left(G^{(k)}, \Gamma^{(k)}, P^{(k)}\right)$$

Convergence Theorem

Theorem

For any activation process, the dynamics initiated at any outcome of the static game converge. Further, if the activation process is a uniform activation process, then the expected number of rounds to convergence is $O(n^5)$.

Given an activation sequence, the limiting state is such that:

1. *the network topology is a tree where any node that is not a leaf is of minimum routing cost.*
2. *It is a pairwise stable outcome of the static game.*

Motivation
Model
Utility
Contracting
Solution Concept
Dynamics
Results
Assumptions
Main Theorems
Efficiency
Interpretation
Conclusion
Future Work

Second Convergence Theorem

During the first stage, exogenously remove the link with some probability.

Then the dynamics converge *even without anti-symmetry*.

Given an activation sequence, the limiting states are such that:

1. the network topology is a tree where any node that is not a leaf is of minimum routing cost.
2. All visited states are pairwise stable outcomes of the static game.

Important Corollary

If there is a unique node of minimum routing cost v_{min}, then the dynamics converge to the star centered at v_{min}.

Any star centered at a node of minimum routing cost minimizes the *price of stability*.

Thus, in this particular case, our dynamics select *the most efficient* pairwise stable outcome.

Motivation

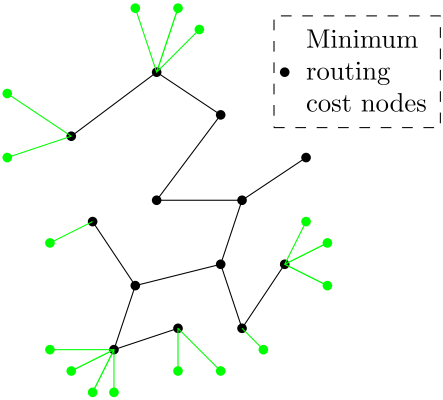
Model
Utility
Contracting
Solution Concept
Dynamics

Results
Assumptions
Main Theorems
Efficiency
Interpretation

Conclusion
Future Work

Selecting "Good" Networks

What happens if we have several nodes of minimum routing cost?



In the limiting state, all traffic is routed by minimum routing cost nodes.

Future Work

We are extending our results to other settings.

- We can generalize the first stage of the dynamics.
- We can constrain the set of possible nodes to a ℓ-neighborhood of the active node.
- Finally, under a reasonable tie-breaking rule, we can assume that $\pi = 0$.

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Model
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Results
Assumptions
Main Theorems
Efficiency
Interpretation

Conclusion
Future Work

Comments? Questions?

Thank you

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- Interpretation

Conclusion
- Future Work